

Algebraic Number Theory

End-semesteral test (back-paper)

INSTRUCTIONS: Total time 3 hours. Solve any **five** problems. Max marks 50, all problems carry equal weight. You may use any result proved in the class without proof.

1. Prove that quadratic fields with discriminant -3 have class number 1.
2. Give an example, with explanation, of a quadratic number field with class number bigger than 1.
3. Determine the torsion part of the unit group of $\mathbb{Q}(\sqrt[3]{7})$.
4. Prove that $\mathbb{Z}[\omega]$ is a PID, where ω is a primitive 6th root of unity in \mathbb{C} .
5. Let \mathbb{A} denote the ring of algebraic integers in \mathbb{C} . Is \mathbb{A} Noetherian? Explain.
6. Let I be a nonzero ideal in the ring of integers in a number field. Show that $||I|| \in I$.